Synthesis of the Decentralized Control System for Robot-Manipulator with Input Saturations

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Abstract—It is discussed the problem of synthesis of the decentralized adaptive-periodic control system for two degrees of freedom robotic manipulator which have an input limitations. The solution of the problem is based on the use of hyperstability criterion, L-dissipativity conditions and dynamic filter-corrector.

Keywords—adaptive control; combined regulator; serial dynamic corrector; hyperstability criterion; input saturation

INTRODUCTION

Control systems for robotic manipulators of various purposes have a special place among the variety of modern automatic system. The relevance of the design and development problems of this class of the control systems is due to a very wide application of robots-manipulators. These devices are used in the metallurgical industry, aviation and automotive industry, chemical manufacturing and other areas [1 – 4]. As a rule, the role of the manipulators is to implement a large number of cyclically repeating process steps and problems of the development of robot control systems are to ensure the high precision tracking of a given movement trajectory. At that in practice the desired movement often has a complicated shape, which is typical for automatic arc welding of the complex compounds, laser or plasma cutting, gluing of the various component products and other. From this point of view manipulators control systems are assumed to be the class of so-called periodic control systems. From this point of view manipulator control systems are assumed to be the class of so-called periodic control systems, the construction of which is expedient to use a specialized block –periodic signals generator [5 – 10].

It should be noted that the robotic manipulators as a control plants are multidimensional (multiply connected) systems with set of input and output signals (each input and output signal corresponds to a single degree of freedom). In several articles which devotes to the development of complicated multiply connected dynamic plants control systems, it is proposed to use the principle of the decentralized control [11 – 13]. This approach lies in the partition of the original multi-dimensional system into several independent or interrelated local subsystems, and the stability analysis of each of them.

It is well known that the development of automatic control systems is associate with a number difficulties. During the operation of practically any control system some of the control plant parameters are subject to change (for example, because of wear of its components or the external influence: changes in temperature, pressure and humidity. These parametric changes could lead to reduce of the system performance. In this context, the control algorithms development should always be carried out considering the level (or the class) of a priori uncertainty of the controlled plant. Moreover, in practice operation of the control objects takes place in the conditions of the continuous action external disturbances, which also must be considered in order to reduce their negative impact on the functioning process of the control system.

One more important peculiarity is what in almost all actual systems there is a so-called input saturation, caused by the necessity of limiting the actuators input signals levels. As applied to the manipulation robots, the need to limit the input signals arises in situations when manipulator links are moved in a closed space and thus the amplitude of the control moments must have the prescribed limits which exclude undesirable robot elements displacement (for example, hitting the manipulator of the constructs that are limit the scope of its movement). It should be noted that taking account of the input limitations when developing of the control systems is a very important requirement, because in the systems which designed without considering of the input saturation in some cases there may be observed the deterioration in quality of their functioning or even loss of working capacity [14].

One of the design techniques of control systems for dynamic plants which operate in the conditions of a priori uncertainty, an external perturbations and input saturation is the use of adaptive regulators. By now it is known quite a wide spectrum of adaptive control systems schemes which are designed taking into account the input saturation [15 – 19]. In particular, in [17 – 19] for single channel plants in the assumption of complete measurement of the state vector and the absence of external noise have been proposed a variety of adaptive control schemes in the circuit with the reference model. The obtained results allowed to achieve the convergence of control error to zero, and boundedness of all signals of the closed system. The main feature of the developed control systems is the introduction of a special regulator switch which is responsible for changing the adaptive coefficients.
adjustment speed, whereby the input saturation is partially or completely compensated. In [20] with the help of hyperstability criterion and L-dissipativity conditions for the scalar plant with the input saturation and the relative order greater than one has been proposed a modification of the adaptive regulator, which allowed to implement effective compensation of limitations on the entry in a non-zero initial conditions, constant external disturbances and unavailability of internal states. In the present article by using the results of [7 – 10, 13, 20 – 24] we discussed the possibility of using a combined regulator structure, as well as proposed in [20] modified adaptive algorithms for the construction of control system for two degrees of freedom robot-manipulator with input saturation.

**INITIAL DESCRIPTION OF THE CONTROL SYSTEM**

The dynamics of the manipulator, which consist of \( n \)-links and having an input saturation, according to [4], is described by

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + f_{du} = S(\tau),
\]

where \( q \in \mathbb{R}^n \) is the vector of coordinates (angular displacement) of the robot links; \( \tau \in \mathbb{R}^n \) is the input control signal (vector of the control torques of each link); \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the matrix describing the centrifugal and Coriolis forces; \( G(q) \in \mathbb{R}^n \) is the vector determining the influence of gravitational forces; \( f_{du} \in \mathbb{R}^n \) is the vector external constantly operating bounded disturbances acting on the manipulator links; \( S(\tau) \) is the non-linear function of the input signal saturation which describes as follows:

\[
S(\tau) = \begin{cases} 
S_0, & \tau > S_0, \\
\tau, & 1 \tau \leq S_0, \\
-S_0, & \tau < -S_0,
\end{cases}
\]

where \( S_0 > 0 \) is the known constant corresponding to limit level.

Let us consider more detail the dynamic properties of the robot manipulator, which includes two degrees of freedom. In accordance with [4] and equation (1) the dynamics of similar object may be presented as follows

\[
\begin{bmatrix}
S(\tau_1) \\
S(\tau_2)
\end{bmatrix} = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix} \begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} + \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix} \begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} + \begin{bmatrix}
G_1 \\
G_2
\end{bmatrix} + \begin{bmatrix}
f_{du,1} \\
f_{du,2}
\end{bmatrix} + \begin{bmatrix}
f_{du,1} \\
f_{du,2}
\end{bmatrix}
\]

The elements of the inertia matrix and matrix of the Coriolis forces have the form

\[
M_{11} = p_1 + p_2 + 2p_3 \cos(q_2); \\
M_{12} = p_1 + p_3 \cos(q_2); \\
M_{21} = p_2 + p_3 \cos(q_2); \\
M_{22} = p_2;
\]

\[
C_{11} = -p_4 \dot{q}_1 \sin(q_2); \\
C_{12} = -p_3 \dot{q}_1 \sin(q_2); \\
C_{21} = p_3 \dot{q}_1 \sin(q_2); \\
C_{22} = 0;
\]

\[
G_1 = p_4 \cos(q_1) + p_3 \cos(q_1 + q_2); \\
G_2 = p_3 \cos(q_1 + q_2);
\]

where

\[
p_1 = m_1 l_1^2 + m_2 l_1^2 + I_1; \\
p_2 = m_1 l_2^2 + I_2; \\
p_3 = m_1 l_1^2 + m_2 l_2^2; \\
p_4 = m_1 l_1^2 + m_2 l_2^2.
\]

In equations (3) – (5) are introduced the following notations: \( q_1, \dot{q}_1, \ddot{q}_1, ... \) are the angular displacement, velocity and acceleration of each link; \( m_i \) is the mass of the \( i \)-th link [kg]; \( I_i \) is the length of the \( i \)-th link [m]; \( I_1 \) is the moment of inertia of each link [kgm²]; \( l_i \) is the distance from \( (i - 1) \)-th connection to the center of mass of the \( i \)-th link [m].

We represent the a mathematical description of the system in vector-matrix form, considering the manipulator, as a multidimensional dynamic plant [13, 22], where each link of the manipulator is assigned its own local subsystem. Introducing the notations: \( \tau = u, \dot{q}_1 = x_1, \ddot{q}_1 = \dot{x}_1, \) \( \dot{x}_1 = x_1 \), \( f_{du,i} = f_i, i = 1, 2; \) and performing a number of obvious mathematical transformations we write the model (3) – c(5) in the state space:

\[
\frac{dx_i(t)}{dt} = A_i(t, x_i(t)) + b_i S(\mu_i(t)) + f_i(t) + b_i \sum_{j=1}^{n} A_{ij}(t, x(t)),
\]

\[
y_i(t) = L^T x_i(t) = x_i(t), i = 1, 2,
\]

where \( x_i(t) = [x_1(t), x_2(t), \ldots, x_i(t)]^T \) is the state variables vector of the local subsystems; \( u(t) \in \mathbb{R} \) are the local control signals; \( y_i(t) \in \mathbb{R} \) is the angular displacement of the robot links (output of the \( i \)-th local subsystem); \( A_i(t, x_i(t)) = A_{ii}(x_i(t), x_i(t))x_i(t) + A_{i*}(x_i(t)) \) is the nonlinear vector function; \( A_i(x_i(t), x_2(t)) = A_i + b_i \alpha_i^T (x_i(t), x_i(t)) \) is the matrix whose elements are the nonlinear functions; \( A_{ii} \) is the some stationary matrix; \( A_{i*} (x_i(t)) = b_i \beta_i (x_i(t)) \) is the some nonlinear vector; \( \alpha_i^T (x_i(t), x_i(t)) \) and \( \beta_i (x_i(t)) \) are the vector and scalar non-linear functions respectively; \( A_{ii}(t, x(t)) \) are the nonlinear functions describing the dynamic interconnection of the robot links; \( f_i(t) \in \mathbb{R}^3 \) is the external permanent disturbances vector, which satisfies the condition
\[ f_j(t) = b_j f_j(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad f_i(t) \leq f_h, \quad f_0 = \text{const} > 0. \] (7)

In accordance with (3) – (5) nonlinear functions that are part of the model (6) and (7) have the form:

for the first degree of freedom

\[
\alpha_i(x_1(t), x_2(t)) = \begin{bmatrix} \alpha_i^1 (x_1(t), x_2(t)) \\ \alpha_i^2 (x_1(t), x_2(t)) \end{bmatrix};
\]
\[
\alpha_i (x_1(t), x_2(t)) = 0;
\]
\[
\alpha_i^1 (x_1(t), x_2(t)) = p_i x_i(t) \sin(x_i(t));
\]
\[
\alpha_i^2 (x_1(t), x_2(t)) = 2 p_i \cos(x_i(t));
\]
\[
\beta_i (x_i(t)) = p_i g \cos(x_i(t));
\]

for the second degree of freedom

\[
\alpha^T_i (x_1(t), x_2(t)) = \begin{bmatrix} \alpha_i^1 (x_1(t), x_2(t)) \\ \alpha_i^2 (x_1(t), x_2(t)) \end{bmatrix};
\]
\[
\alpha_i^1 (x_1(t), x_2(t)) = 0;
\]
\[
\alpha_i^2 (x_1(t), x_2(t)) = 0, \quad \beta_i^2 (x_2(t)) = 0.
\]

the dynamic cross-connections

\[
\lambda_i (t, x(t)) = (p_2 + p_1 \cos(x_i(t))) x_i(t) - (p_3 x_1(t) + x_2(t)) \sin(x_1(t)) + p_3 g \cos(x_1(t)) x_2(t) + p_4 g \cos(x_1(t)) x_2(t) + (p_3 \sin(x_1(t))) x_1(t) + p_4 g \cos(x_1(t)) x_2(t).
\] (10)

To specify the required movement of the robot links, similar to [20], we use the local reference models with two outputs:

\[
\frac{dx_{M_i}}{dt} = A_{M_i} x_{M_i}(t) + B_{M_{ij}} r_j(t),
\]
\[
y_{M_i}(t) = x_{M_{ij}}(t), \quad z_{M_i}(t) = g^T_{M_i} x_{M_i}(t), \quad i = 1, 2,
\]

where \( x_{M_i}(t) = [x_{M_{1i}}, x_{M_{2i}}, x_{M_{3i}}]^T \) is the state vector of the local reference models; \( B_{M_{ij}} = [0, 0, b_{M_{ij}}]^T, \quad b_{M_{ij}} = \text{const} > 0; \)
\( r_j(t) = r_j(t + T) \in R \) is the scalar periodic command signal;

\( y_{M_i}(t) \in R, \quad z_{M_i}(t) \in R \) are the main and auxiliary outputs of the reference model; \( g_i \) is the given vector; \( A_{M_i} = (A_i - B_{M_{ij}} c_{M_{ij}}) \) is the Hurwitz matrix in the Frobenius form; \( c_{M_i} = [c_{M_{0i}}, c_{M_{1i}}, c_{M_{2i}}] \) is the vector with given numbers.

The regulator structure will be given as the following adaptive-periodic combination

\[
u_i(t) = k_i \theta_i(t) - c_i^T(t) x_i(t), \quad i = 1, 2; \] (12)

where \( k_i = \text{const} > 0; \) \( \theta_i(t) \) is the periodic signals generator output (regulator periodic setting); \( c_i(t) \) is the vector of self-tuning coefficients (regulator adaptive setting).

THE PROBLEM STATEMENT

For multidimensional dynamic plant (3) – (10) with using the reference model (11) it is necessary to synthesize the explicit form of the self-tuning algorithms for vector of bootstrapping coefficients \( c_i(t) \), providing for any initial condition \( x(0) \) and any changes of the functions \( \alpha_i(x_i(t), x_2(t)) \), \( \lambda_i(t, x(t)) \) an implementation of limit target conditions

\[
\lim_{t \to \infty} \| c_i(t) \| \leq c_{0i}, \quad c_{0i} = \text{const} > 0,
\] (13)

\[
\lim_{t \to \infty} y_{M_i}(t) - y_i(t) \leq y_{0i}, \quad y_{0i} = \text{const} > 0, \quad i = 1, 2.
\] (14)

where \( c_{0i}, \quad y_{0i} \) are sufficiently small numbers.

SOLUTION METHOD, ALGORITHMS OF THE CONTROL LOOP AND L-DISSIPATIVITY CONDITIONS OF THE CONTROL SYSTEM

To solve this problem we use the same method proposed in [20, 21], namely:

Under the assumption of the availability of all state variables \( x(t) \) using hyperstability criterion we will determine the explicit form of the self-tuning algorithms for coefficients \( \theta_i(t) \) and \( c_i(t) \) of the combined regulator (12).

For the technical implementation of synthesized at the previous step control algorithms, we will introduce in each subsystem filter-corrector to obtain estimates of the state vectors \( x_i(t) \) and define the specific conditions to ensure the operability and L-dissipativity of the developed system.

According to [7 – 10, 20], in the case of availability of the internal states of local subsystems (6) – (10) by using the hyperstability criterion we can show that the synthesis of the regulator (12) algorithms in the form

\[
\theta_i(t) = \theta_i(t - T_i) + \nu_i(t), \quad \nu_i(s) = 0, \quad s \in [-T_i; 0],
\] \[ T_i = \text{const} > 0, \] (15)
\[
\begin{align*}
dc(t) = & \begin{cases} 
-h_k x_i(t)v_i(t)\bar{\delta}(t), & \forall 1 v_i(t) > 0, \\
0, & \forall 1 v_i(t) \leq 0,
\end{cases} \\
h_k = \text{const} > 0, & c_k(0) = 0, \quad k = 1, 2, 3, \quad i = 1, 2.
\end{align*}
\]

where \( v_i(t) = z_i(t) - z(t) \) are the mismatch signals of the auxiliary outputs of the local reference models (11) and the control plant subsystems (6) – (10) formed by using the vectors \( g_i; \phi_i = \text{const} > 0 \) are values of the dead zones; \( \bar{\delta}(t) \) are outputs of the local dynamic switches

\[
\begin{align*}
d_i \frac{d\bar{\delta}(t)}{dt} + \bar{\delta}(t) = \delta_i(t), \quad d_i = \text{const} > 0, \\
\delta_i(t) = \begin{cases} 
1, & \forall [S(u_i(t)) - u_i(t)]v_i(t) \geq 0, \\
\delta_i, & \forall [S(u_i(t)) - u_i(t)]v_i(t) < 0, \\
0 < \delta_i = \text{const} < 1,
\end{cases}
\end{align*}
\]

will ensure the hyperstability and adaptability of the system (6) – (12), (15) – (17) and perform for it the targets (13), (14).

Since local variables of the dynamic plant (6) – (10) is not available for measuring we connected to the output of each local subsystem a filter-corrector, which is described by equations

\[
\begin{align*}
\frac{dx_F(t)}{dt} = & A_F x_F(t) + B_F y_i(t), \\
z_F(t) = & g_F T x_F(t) + D_F y_i(t), \\
W_F(s) = & \frac{z_F(s)}{y_i(s)} = \frac{g_F(sE_F - A_F)^{-1}B_F}{\det(sE_F - A_F)} + D_F = \\
& = \frac{g_F(s)}{(T_F s + 1)^2};
\end{align*}
\]

where \( z_F(t) \in R \) and \( x_F(t) = [x_{F1}(t), x_{F2}(t)]^T \) are the scalar outputs and filters state vectors respectively; \( T_F \) are the small time constants; \( W_F(s) \) are filter transfer functions; \( s \) is the complex variable.

In this case, we can show [20, 21] that, when considering a certain steady state of the multiply connected system (6) – (12), (15) – (18) local contours while setting the value of the parameter \( T_F \) based on the conditions

\[
T_F < T_1 = \frac{0.93}{c_{M1}}, \quad T_F < T_2 = \frac{0.465e_{M1}}{2e_{M2}};
\]

it is possible to ensure \( L \)-dissipativity of the system, which losing a hyperstability will retain operability and adaptability in a given class. Herewith the local adaptive regulators are transformed to the form

\[
u_i(t) = k_i \theta_i(t) - c_{i}^T(t)x_F(t), \quad i = 1, 2; \quad \theta_i(t) = \phi_i(t - T_F) + \bar{v}_i(t), \quad \theta(s) = 0, \quad s \in [-T_F; 0].
\]

\[
\frac{dc_i(t)}{dt} = \begin{cases} 
-h_k x_i(t)\bar{v}_i(t)\bar{\delta}(t), & \forall 1 \bar{v}_i(t) > 0, \\
0, & \forall 1 \bar{v}_i(t) \leq 0,
\end{cases} \\
h_k = \text{const} > 0, & c_k(0) = 0, \quad k = 1, 2, 3, \quad i = 1, 2;
\]

where \( x_F(t) = [x_{F1}, x_{F2}, \dot{x}_{F2}]^T \) are estimates of the local subsystems (6) state variables.

**COMPUTATIONAL EXPERIMENT**

Let us consider control problem of the two degrees of freedom manipulator with input saturations and dynamics, which describes by equations

\[
\begin{align*}
S(\tau_1) = & \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \bar{\bar{q}}_1 + \\
+ & \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \bar{\bar{q}}_1 + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} + \begin{bmatrix} f_{d1} \\ f_{d2} \end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
M_{11} = & p_1 + 2p_1 \cos(q_1); \quad & M_{12} = p_2 + p_3 \cos(q_2); \\
M_{21} = & p_2 + p_3 \cos(q_2); \quad & M_{22} = p_2; \\
C_{11} = & -p_1q_1 \sin(q_1); \quad & C_{12} = -p_1(q_1 + q_2) \sin(q_2); \\
C_{21} = & p_1q_1 \sin(q_1); \quad & C_{22} = 0; \\
G_1 = & p_2g \cos(q_1) + p_3 \cos(q_1); \quad & G_2 = p_2g \cos(q_1 + q_2); \\
p_1 = & m_1l_1^2 + m_2l_2^2 + I_1; \quad & p_2 = m_2l_2^2 + I_2; \\
p_3 = & m_1l_2 + m_1l_1; \quad & p_5 = m_2l_1^2.
\end{align*}
\]

where \( q_i, \dot{q}_i, \ddot{q}_i, i = 1, 2 \) are the angular displacement, velocity and acceleration of each link; \( m_i \) is the mass of the first and the second link of the manipulator [kg]; \( l_i \) is the length of the \( i \)-th link [m]; \( l_i \) is the moment of inertia of each link [kg·m²]; \( l_i \) is the distance from \((i-1)\)-th connection to the center of mass of the \( i \)-th link [m].

For the experiment we take the robot parameters as follows:

\[
\begin{align*}
m_1 = & 1.2 \text{ kg}; \quad m_2 = 0.8 \text{ kg}; \quad l_1 = 0.35 \text{ m}; \quad l_2 = 0.31 \text{ m}; \quad l_1 = \frac{61.25 \times 10^{-3}}{20.42 \times 10^{-3}} \text{ kg·m²}; \quad l_2 = 20.42 \times 10^{-3} \text{ kg·m²}.
\end{align*}
\]

External disturbances acting on the considered control plant are described as follows:

\[
f_{d1}(t) = 0.1 \cdot (\sin(\pi t) + 2 \sin(2\pi t) + 3 \sin(3\pi t)),
\]

\[
i = 1, 2.
\]
To the outputs of local subsystems we connect the filter correctors
\[
\dot{x}_{F1}(t) = x_{F2}(t); \\
\dot{x}_{F2}(t) = -10^6 x_{F1}(t) - 2000 x_{F2}(t) + 10^6 y_i(t); \\
z_F(t) = x_{F1}(t) + x_{F2}(t) + 0.25 \dot{x}_{F2}(t); \quad x_{F1}(0) = x_{F2}(0) = 0;
\]
and their desired dynamics and the dynamics of the decentralized system local main circuits we specify by explicit etalons
\[
\dot{x}_{M1}(t) = x_{M2}(t); \\
\dot{x}_{M2}(t) = x_{M3}(t); \\
\dot{x}_{M3}(t) = -8x_{M1}(t) - 12x_{M2}(t) - 6x_{M3}(t) + 8r_i(t); \\
y_{M1}(t) = x_{M1}(t), \quad y_{M2}(t) = x_{M1}(t) + x_{M2}(t) + 0.25x_{M3}(t); \\
x_{M1}(0) = x_{M2}(0) = x_{M3}(0) = 0; \quad r_i(t) = 1.25 \sin(t).
\]

During the simulation input constraints of the robot links actuators were set with the values \( S_1 = 7.1 \) and \( S_2 = 5 \). Parameters of the combined control circuit (9) – (12) in order to increase system performance were selected as follows:
\[
k_1 = 300; \quad h_{h1} = 100; \quad h_{h2} = 20; \quad h_{h3} = 1; \quad \phi_1 = 0.001; \\
k_2 = 1000; \quad h_{h1} = 750; \quad h_{h2} = 500; \quad h_{h3} = 10; \quad \phi_2 = 0.001; \\
\delta_{\theta_1} = 0.0007; \quad d_1 = 5; \quad \delta_{\theta_2} = 0.0006; \quad d_2 = 5; \quad T_1 = 2; \quad T_2 = 2.5.
\]

In Fig. 1 – 3 are presented the results of system operation when the initial positions of the manipulator links are \( y_1(0) = -1.2 \) and \( y_2(0) = 1.2 \).

Presented results are indicate a sufficiently good quality of functioning of the developed system.

**CONCLUSION**

It is presented the solution of the control problem of a robot manipulator with two degrees of freedom with input saturation. By using the hyperstability criterion, L-dissipativity conditions and dynamic filter-corrector it is constructed the decentralized control system with combined adaptive-periodic local control circuits.

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