

Analysis of Webspaces of the Siberian Branch of the Russian Academy of Sciences and the Fraunhofer-Gesellschaft

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Abstract—In this paper, two webspaces of academic institutions of the Siberian Branch of Russian Academy of Sciences (SB RAS) and of the Fraunhofer-Gesellschaft (FG), Germany, will be investigated. The webspaces are represented by directed graphs possessing vertices corresponding to websites. An arc connects two vertices if there exists at least one hyperlink between the corresponding websites. Webometrics is used for ranking the websites of SB RAS and FG. We discuss numerical results when studying the websites structurally. In particular, we examine scientific communities of the underlying websites representing directed graphs and draw important conclusions.

Keywords—network; webometrics; quantitative measure; communities

I. INTRODUCTION

In this paper, a webspace is a structural object (graph) formed by a set of websites and hyperlinks between them [1]. To investigate a webspace structurally, we use methods from webometrics, i.e., the contemporary method for studying information resources, structure and technology features of the web. The development of webometrics has started in 1997 after the seminal paper of Almind and Ingwersen [2]. Methods from webometrics possess statistical nature and do not serve as full description of diverse information processes that occur in the webspace. Therefore to analyze the structure of webspaces, we are going to use graph-theoretical methods [1, 3].

There have been a large number of contributions in the literature for studying webspaces resembling university websites and academic institutions [4-12]. Since the number of webspaces to be studied is infinite, there is still space left for performing research on this topic. In this paper, we tackle this problem by studying websites from Russia to investigate the underlying institutions specifically.

It is well-known that the structure of real-life networks is not random [13]. When dealing with non-random topologies, the

structural heterogeneity (or complexity) may be captured by calculating various measures [14, 15]. Another problem in this context is to determine the community structure of networks. Community detection has been one of the hot topics in network sciences and, hence, the problem received considerable attention in the last decade [16]. The concept of the community in a network is usually derived from common understanding of communities in social networks [17]. Graph-theoretically, the problem has been defined by identifying the set of vertices which are more tightly connected compared to the rest of the network. Note that in order to solve the community problem, a precise mathematical quantity Q has been introduced based on the following description: to partition the vertex set of a network into a union of subsets that maximizes Q [18]. However, this problem has been proven to be NP-hard and only heuristics algorithms are available to determine Q [19].

Here we consider webspaces generated by websites of academic institutions of the Siberian Branch of the Russian Academy of Sciences (SB RAS) and academic institutions of the Fraunhofer-Gesellschaft (FG), Germany. The structure of these webspaces is formed by websites of the scientific institutions and hyperlinks between them. Websites of SB RAS and FG will be ranked by using methods from webometrics, and numerical scores for examining community structure of webspaces are determined. Since Russian webspaces have only been little investigated, we believe that our work will have an impact for the webscience community.

II. REPRESENTATION OF WEBSPACES

A simple model for representing the structure of webspaces is a directed weighted graph $G = (V, E)$ with vertex set V and arc set E . We assume that webgraphs [14] do not possess any self-loops and multi-arcs. In this paper, vertices of V correspond to websites. Suppose that the vertices v and u of G correspond to sites X and Y ; then an arc (v, u) connects the vertices v and u if

there exists at least one hyperlink in X , referring to site Y . The number of hyperlinks from X to Y is represented by the weight w of the arc (v,u) . The distance $d(v,u)$ between vertices v and u in a graph is the number of arcs in the shortest directed path connecting them.

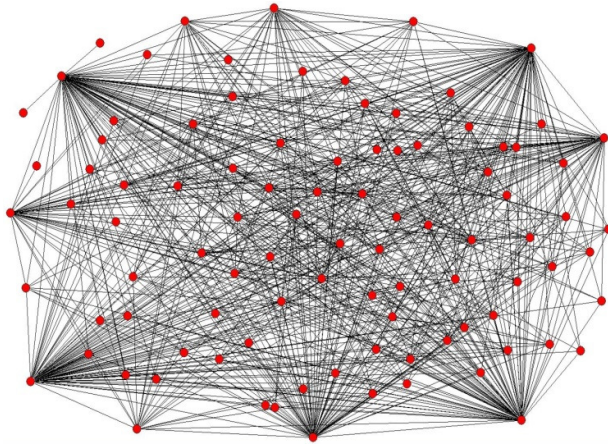


Fig. 1. Webgraph R of institutes of Siberian Branch of RAS.

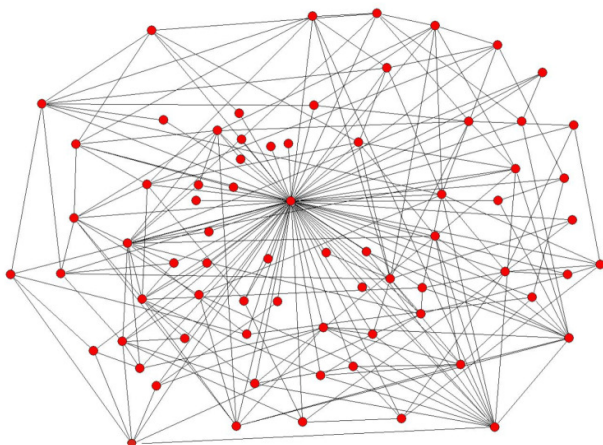


Fig. 2. Webgraph F of institutes of Fraunhofer-Gesellschaft.

Let R and F be webgraphs of SB RAS and FG, respectively. Their structures are shown in Fig. 1 and Fig.2. Graph R has 95 vertices and 949 arcs while G consists of 72 vertices and 321 arcs. Additional information on these graphs can be found in [20,21]. The number of arcs going from (to) a vertex v is denoted by $deg^+(v)$ (in-degree $deg^-(v)$). A pair $(deg^+(v), deg^-(v))$ gives information on vicinity size of v . The weighted out-degree (in-degree) $wdeg^+(v)$ ($wdeg^-(v)$) is the sum of weights of arcs coming from (reaching) a vertex v . Total degrees are defined as $deg(v) = deg^+(v) + deg^-(v)$ and $wdeg(v) = wdeg^+(v) + wdeg^-(v)$. A vertex v is called isolated if $deg(v) = 0$. The degree distributions of the graphs R and F are shown in Fig. 3 to Fig. 6.

III. METHODS

A. Ranking academic institutions by using webometrics

The “Ranking Web of World Research Centers” is an initiative of the Cybermetrics Lab, a research group belonging to

the Consejo Superior de Investigaciones Científicas (CSIC), Spain. Quantitative methods have been designed to measure the scientific activity on the Web to determine ratings of universities and research centers of various countries [22]. The cybermetric indicators have been useful to evaluate science and technology and they serve as a proper complement to the results obtained by using bibliometric methods connected to scientometric studies.

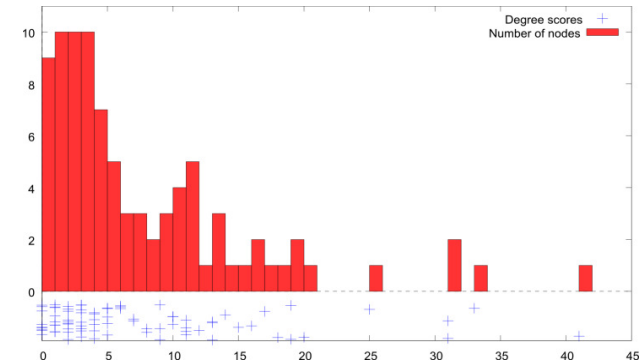


Fig. 3. Degree distribution of graph R .

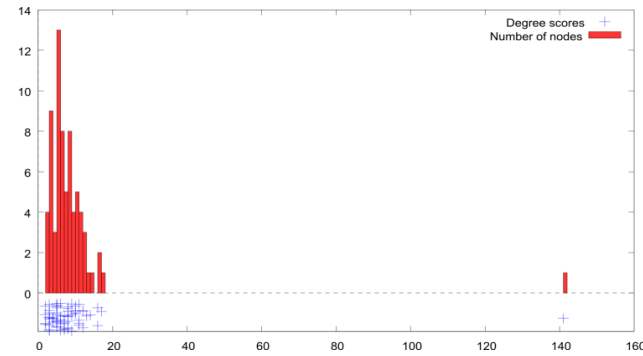


Fig. 4. Degree distribution of graph F .

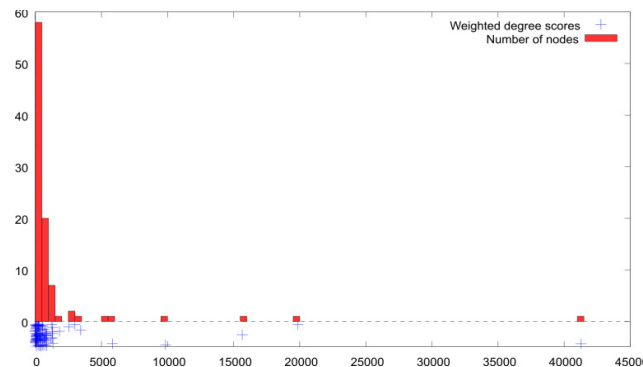


Fig. 5. Weighted degree distribution of graph R .

Starting from 2008, the Institute of Computational Technologies of Siberian branch of Russian Academy of Sciences (SB RAS) generates ratings of websites of scientific institutions of SB RAS [6, 8, 21]. The ranking method is presented in [22]. In this paper, we are going to extract statistics from three major search engines: Yandex [23], Google [24], and Bing [25]. To evaluate websites, the method uses the following parameters:

- **V** – visibility. The parameter equals the number of external links from other websites to the considered one. Since the data from different engines is distinct, the average value is taken: $\mathbf{V} = (\mathbf{V}_{\text{Yandex}} + \mathbf{V}_{\text{Google}} + \mathbf{V}_{\text{Bing}})/3$.
- **S** – size. The parameter equals the number of webpages of the website determined by the search engines. Again, we use the average value: $\mathbf{S} = (\mathbf{S}_{\text{Yandex}} + \mathbf{S}_{\text{Google}} + \mathbf{S}_{\text{Bing}})/3$.
- **R** – richness value. The parameter equals to the number of documents the website has with file extensions of Adobe Acrobat (.pdf), Microsoft Word (.doc) and PowerPoint (.ppt). The quantity is determined by search engines' query, therefore we use averaging: $\mathbf{R} = (\mathbf{R}_{\text{Yandex}} + \mathbf{R}_{\text{Google}})/2$.
- **Sc** – citation index obtained from citation system Google Scholar [26]. This parameter reflects the academic importance of the website.

The overall rating evaluation includes the following steps.

1. Evaluation of the visibility **V**, size **S** and richness **R** parameters for all websites in the network.

2. Ranking the values of **V**, **S** and **R**. The parameter array, say **V**, is ranked in decreasing order. The website with maximal **V** receives rank $\mathbf{V}_r = 1$. The websites with identical values of **V** get equal ranks. Similarly, we compute the ranks \mathbf{S}_r and \mathbf{R}_r by using the parameters **S** and **R** for each website in the network.

3. Evaluation of the rank of the citation index **Sc**. We compute the values **Sc**. The rank \mathbf{Sc}_r is obtained by ordering these values. The website with the minimal value receives the rank-value $\mathbf{Sc}_r = 1$.

4. Computing the sum of the obtained ranks for each website: $\mathbf{W} = \mathbf{V}_r + \mathbf{S}_r + \mathbf{R}_r + \mathbf{Sc}_r$.

5. The final rating is obtained by sorting the list of **W** scores in increasing order. Therefore, the lower the value of **W** is, the higher is the rank (rating position) of the website.

B. Quantitative measures for webgraphs

One of the common approaches when studying web structures is based on quantifying structural information by using various quantitative measures [14, 15]. Usually, a quantitative graph measure is a graph invariant that maps a set of graphs to a set of numbers such that invariant values coincide for isomorphic graphs [27]. Such invariants can quantify either local or global properties of graphs. Local measures, as a rule, describe a graph structure near particular vertices. In contrast, global measures encode structural information of the entire graph. Some global invariants may be regarded as a complexity measure of a graph [28, 29]. We consider the following graph invariants.

The average degree, $adeg(G)$, of a n -vertex graph G is the average value:

$$adeg(G) = \frac{1}{n} \sum_{v \in V(G)} deg^+(v) = \frac{1}{n} \sum_{v \in V(G)} deg^-(v).$$

The weighted analogue, $awdeg(G)$, is given by the formula:

$$awdeg(G) = \frac{1}{n} \sum_{v \in V(G)} wdeg^+(v) = \frac{1}{n} \sum_{v \in V(G)} wdeg^-(v).$$

The diameter, $diam(G)$, of a graph G is the largest distance between two vertices:

$$diam(G) = \max\{d(u, v) \mid u, v \in V(G)\}.$$

It says how far one can travel in a webspace without any repetitions of websites.

The vertex index, $c_v(G)$, of a graph G . This invariant indicates which part of a website is involved into information relationships (every website of this part has at least one arc). Let G be a n -vertex graph with k isolated vertices. Then

$$c_v(G) = 1 - \frac{k}{n}.$$

The quantity $c_v(G)$ reflects stages of webspace growth. Namely, $c_v(G)$ is close to 0 in the initial stage when forming the webspace; the value $c_v(G) = 1$ indicates that all websites are contained in the network.

The arc index, $c_a(G)$, of a graph G . The maximal number of arcs in a directed n -vertex graph is equal to $n(n-1)$, $n > 1$. Let G has t arcs. Then the arc index is defined as

$$c_a(G) = \frac{t}{n(n-1)}.$$

This graph invariant is also referred to as network density [30]. The quantity $c_a(G)$ shows which part of arcs participate in changes between websites. The maximal value $c_a(G) = 1$ expresses that one can reach any other website by one click starting from an arbitrary website.

The betweenness centrality, $betw(v)$, of a vertex v shows the importance of a vertex in terms of routing and connectivity. This quantity [31] is a local graph invariant defined as:

$$betw(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}},$$

where σ_{st} is the total number of directed shortest paths from vertex s to vertex t and $\sigma_{st}(v)$ is the number of those paths that pass through v .

The clustering coefficient, $cc(G)$, of a graph G . By writing neighborhood of a vertex v , we refer to all vertices that are adjacent to v (without orientation of arcs). Let V_2 be the set of all vertices of a directed graph G with $deg(v) = 2$. Let G_v be the directed subgraph induced by the neighborhood of v . The clustering coefficient for a vertex v is defined by $c_a(G_v)$, i.e. it is the arc index of G_v [17, 32]. Then the clustering coefficient of G is the average value of the clustering coefficients for all vertices regarding V_2 , namely:

$$cc(G) = \frac{1}{|V_2|} \sum_{v \in V_2} c_a(G_v).$$

The introduced numerical graph invariants are applied to the web graphs R and F .

C. Communities in graphs

We study the community structure in terms of splitting the vertex set V into non-intersecting subsets (or communities) that maximize the directed and weighted modification of modularity coefficient [18]. Denote by w_{ij} the weight of an arc (i, j) of a graph G with vertex set $V = \{1, 2, \dots, n\}$. For weighted degrees of vertices and the total degree w , we get $wdeg^+(i) = \sum_{(i,j)} w_{ij}$, $wdeg^-(i) = \sum_{(j,i)} w_{ij}$ and $w = \sum_i wdeg^+(i) = \sum_i wdeg^-(i)$. Then the modularity $Q(G)$ can be defined as

$$Q(G) = \frac{1}{w} \sum_{(i,j) \in E} \left(w_{ij} - \frac{wdeg^+(i) wdeg^-(j)}{w} \right) \delta(C_i, C_j),$$

where C_i is the cluster of vertex i and $\delta(C_i, C_j)$ is the Kronecker symbol. It equals 1 if the vertices i and j are in the same community; otherwise it equals 0. The unweighted version of modularity, $Q_{un}(G)$, is obtained from $Q(G)$ by omitting the weight from every arc. That is for every arc (i, j) we assign a new weight $w_{ij}' = 1$ if $w_{ij} \neq 0$ and $w_{ij}' = 0$ otherwise. If a graph G has q arcs, then $Q_{un}(G)$ can be written as follows [18, 33]:

$$Q_{un}(G) = \frac{1}{q} \sum_{(i,j) \in E} \left(1 - \frac{deg^+(i) deg^-(j)}{q} \right) \delta(C_i, C_j).$$

The quantities Q and Q_{un} are applied to the graphs R and F .

IV. RESULTS AND DISCUSSION

A. Ranking academic institutions

The results from final ranking the websites academic institutions of Siberian Branch of RAS and Fraunhofer-Gesellschaft are presented in [20, 21]. First parts of the rankings are shown in Table I and Table II. The comparison of four webometrics quantities for these websites is presented in Fig. 7 and Fig. 8. From these rankings and involved computations (V , S , R and Sc) we are able to perform several observations:

TABLE I. RATING SCORES P FOR INSTITUTIONS OF SB RAS

P	Name of organization	Website address	W
1	Portal of Siberian Branch of Russian Academy of Sciences	www.sbras.ru	7
2	Institute of Computational Technologies	www.ict.nsc.ru	22
3	Institute of Cytology and Genetics	www.bionet.nsc.ru	22
4	Budker Institute of Nuclear Physics	www.inp.nsk.su	27
5	Sobolev Institute of Mathematics	www.inp.nsk.su	35
6	Institute Computing Simulation	icm.krasn.ru	35
7	State Public Scientific Technological Library	www.spsl.nsc.ru	42
8	A.P. Ershov Institute of Informatics Systems	www.iis.nsk.su	44
9	Branch of SPSL SB RAS	www.prometeus.nsc.ru	51
10	Institute of Automation and Electrometry	www.iae.nsk.su	53
11	Institute of Problems of Development of the North	www.ipdn.ru	56
12	Novosibirsk Institute of Organic Chemistry	www.nioch.nsc.ru	60
13	Boreskov Institute of Catalysis	www.catatysis.ru	61
14	Presidium of SB RAS	www.sbras.nsc.ru	68
15	Kirensky Institute of Physics	www.kirensky.ru	70

TABLE II. RATING SCORES P FOR INSTITUTIONS OF FG

P	Name of organization	Website address	W
1	Fraunhofer Headquarters	www.fraunhofer.de	6
2	Institute for Systems and Innovation Research	www.isi.fraunhofer.de	26
3	Institute for Open Communication Systems	www.fokus.fraunhofer.de	30
4	Institute for Manufacturing Engineering and Automation	www.ipa.fraunhofer.de	34
5	Institute for Industrial Mathematics	www.itwm.fraunhofer.de	37
6	Institute for Solar Energy Systems	www.ise.fraunhofer.de	42
7	Institute for Industrial Engineering	www.iao.fraunhofer.de	43
8	Institute for Laser Technology	www.ilt.fraunhofer.de	43
9	Institute for Integrated Circuits	www.iis.fraunhofer.de	46
10	Institute for Information Center for Planning and Building	www.irb.fraunhofer.de	59
11	Institute for Factory Operation and Automation	www.iff.fraunhofer.de	62
12	Institute for Algorithms and Scientific Computing	www.scai.fraunhofer.de	72
13	Institute for Building Physics	www.ibp.fraunhofer.de	75
14	Institute for Intelligent Analysis and Information Systems	www.iais.fraunhofer.de	78
15	Institute for Wind Energy and Energy System Technology	www.iwes.fraunhofer.de	82

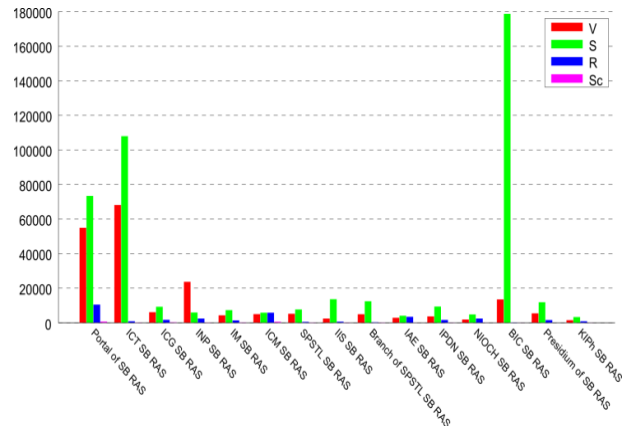


Fig. 7. Final rating scores of the first 15 institutions of SB RAS.

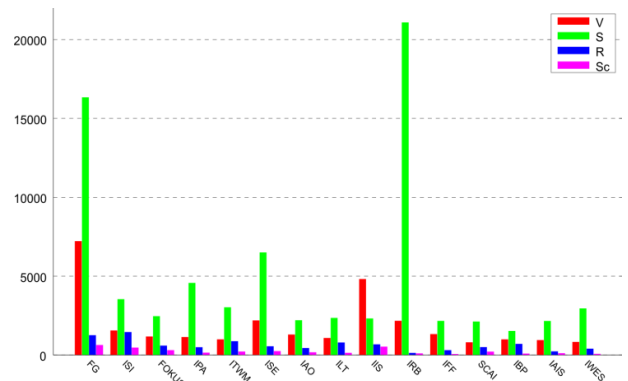


Fig. 8. Final rating scores of the first 15 institutions of FG.

- 27 websites of the SB RAS network and 18 websites of the FG network have more than 1000 external links.

Therefore, 28% of websites of the SB RAS network and 25% of websites of the FG network have sufficiently many external links;

- 88% of the websites of the SB RAS network and 95% of the websites of the FG network have more than 100 webpages. The composition of the websites of the SB RAS and FG networks is similar: the number of websites for SB RAS with $R > 100$ is 47 (45%); for FG we obtain 48 (35%);
- the Google Scholar citation index for FG websites is greater than for websites of the SB RAS network: the number of websites with parameter S_c exceeding 10 is 42 (44%); for SB RAS and 66 (92%) for FG.

B. Quantitative graph measures

Global quantitative properties of the considered graphs are presented in Table III. The average vertex degrees of the graphs decrease twice while the weighted average degrees decrease ten times after deleting the administrative hubs. The diameter of the graphs ranges from 2 in R and for F to 7, respectively.

The vertex index c_v indicates that three graphs contain isolated vertices, i.e., the corresponding websites aren't involved in any communications. That means nobody can neither visit these websites, nor leave them. Namely, graph R has 2 isolated vertices. The other invariants of Table III show that almost all global and local arc saturations of webgraphs are very small.

TABLE III. QUANTITATIVE INVARIANTS OF WEBGRAPHS

Invariant	R (SB RAS)	F (FG)
average degree, $adeg(G)$	9.99	4.46
average degree, $awdeg(G)$	743.21	763.88
graph diameter, $diam(G)$	2	2
vertex index, $c_v(G)$	0.98	1.00
arc index, $c_a(G)$	0.11	0.06
transitivity coefficient, $cc(G)$	0.07	0.09

A local invariant, the betweenness centrality, shows the webgraphs are very centralized. Approximately 8% of vertices possess a significantly higher betweenness centrality score and degrees comparing to the rest. The betweenness centrality scores are shown in Fig. 9 and Fig. 10. Some structural properties of webgraphs R and F have been studied in [9].

V. DISCUSSION AND CONCLUSION

The search of modularity maxima has been performed by using a combination of heuristic algorithms, mainly based on the tabu search algorithm [33]. The observed heterogeneity expressed by vertex degrees and the betweenness centrality score makes it difficult to reveal communities in the network. The best obtained modularity score was ≈ 0.15 for R and ≈ 0.13 for F (see Table IV). Whenever the algorithm assigns a community to one of the most influential vertex, its neighborhoods showed a tendency to fall into this community. The numbers of the corresponding communities and their sizes are presented in Table V.

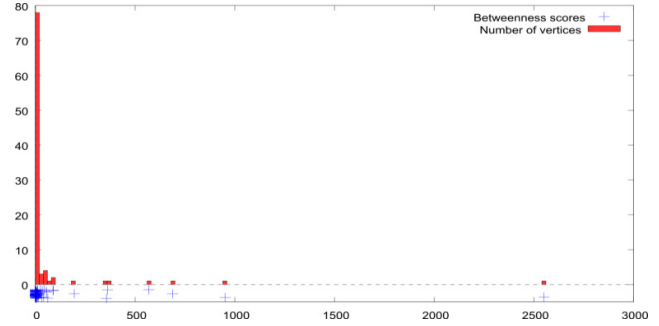


Fig 9. Betweenness centrality distribution of graph R .

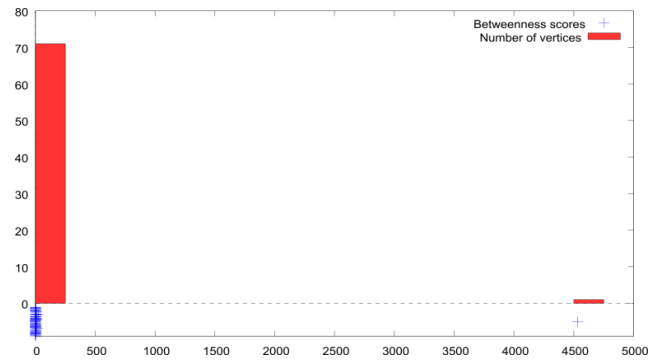


Fig 10. Betweenness centrality distribution of graph F .

TABLE IV. MODULARITY RANKS

	R (SB RAS)	F (FG)
modularity Q	0.153	0.131
modularity Q_{un}	0.155	0.252

TABLE V. NUMBER OF COMMUNITIES AND THEIR SIZES

	R (SB RAS)	F (FG)
Q	5 (46,41,6,1,1)	5 (62,4,2,2,2)
Q_{un}	8 (30,18,14,11,10,10,1,1)	7 (21,10,10,9,9,4)

Along with the weighted modularity, we also computed the unweighted version. The unweighted graph showed a weaker community structure when considering R compared to F . We emphasize that by omitting the weights, we got better partitions (see Table IV and Table V).

It is evident assuming that the communities in these academic networks should reflect scientific collaborations between the corresponding institutes. This hypothesis has been checked for the SB RAS graph R , where we composed the found partitions into communities based on the subject areas of institutes. The resulting subject partition has modularity rank $Q = 0.115$ for R which is far from the optimally obtained partitions.

ACKNOWLEDGMENT

This work was supported in part by the Austrian Science Funds for supporting this work (project P26142), the Russian Foundation for Basic Research (grant 16-01-00499), Presidium RAS (grant 0314-2015-0011), the Leading Scientific Schools of the Russian Federation (grant 7214.2016.9).

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